Approximate Transient Analysis of an Squirrel Cage Induction Motor under DC Dynamic Braking

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Abstract: Transient Analysis of Electrical machines is the cynosure of attention and the subject of utmost interest of most machine designers in their quest to improve machine reliability and achieve the perfect design. Stator being the static part of an induction machine is the seat of higher temperatures, Hence is more liable to thermal failure than the rotor. Thus the study of thermal profile of the stator becomes more critical to identify the causes of failure in an induction machine. This paper presents a two dimensional transient heat flow in the stator using arch shaped elements in the$\boldsymbol{r-\theta}$ ɵ plane of the cylindrical co-ordinate system of an 11 KW TEFC Squirrel Cage induction motor under the condition of DC dynamic braking for different values of stator currents injected in the stator of the aforesaid motor.

Keywords: Induction Motor, FEM, Temperature rise, Insulation, Thermal Analysis, Transient, Design Performance.

Introduction

The thermal analysis of squirrel cage induction motor and the prediction of temperature rise within the magnetic core , insulation and conductors etc. is basically concerned with the determination of three dimensional temperature distribution is produced by an intricate system of current carrying conductors which act as heat sources. The complexity and perplexity of a three dimensional problem can be reduced to a more simplified and tractable two-dimensional problem by a very simple assumption that two independent co-ordinates are required to describe or characterize the geometry of the cylindrical arch-shaped element. For the problems of such nature we assume a arch shaped element or any other relevant geometry of unit thickness in the$r-\theta$ɵ plane of the cylindrical co-ordinate system.

The early designers and researchers have resorted to primitive methods like conformal mapping, resistance analog network etc to determine machine temperatures .These methods suffer from major disadvantages and are based on too many aspersions and assumptions which can raise questions on their veracity. Methods of a more popular and veracious nature are the finite difference and the finite element method , while the former can provide an estimate of copper and iron winding temperatures of the electrical machine , it is not as flexible as the latter. Use of finite element methods has been in vogue to predict transient as well as steady state temperatures among some researchers and is an established numerical analysis procedure for one seeking to find the thermal profile of an electrical machine.

Traditionally, thermal studies of electrical machines have been carried out by analytical techniques, or by thermal network method [1], [2]. These techniques are useful when approximations to thermal circuit parameters and geometry are accepted. Numerical techniques based on either finite difference method [3], [4] or finite element methods [5]–[12] & [15]-[18] are more suitable for analysis of complex system. Rajagopal, M.S, Kulkarni, D.B, Seetharamu, K.N, and Ashwathnarayana P.A [13, 14] have carried out two-dimensional steady state and transient thermal analysis of TEFC machines using FEM. Compared to the finite difference method finite element method can easily handle complicated boundary configurations and discontinuities in material properties.

The finite element method is first introduced for the steady state thermal analysis of the stator cores of large turbine-generators by Armor and Chari [7]. However, their works are restricted to core packages far from the ends and they do not consider the influence of the stator coil heat. In 1980, Armor [9] employed arch-shaped finite elements to solve the transient heat flow in the rotor of large turbine-generators. Sarkar and Bhattacharya [15] also described a method based on arch-shaped finite elements with explicitly derived solution matrices for determining the thermal field of induction motors.

However the finite element method has sparsely been attempted mostly because of its esoteric nature and computational complexity, use of finite element to solve the temperature distribution of the machine during DC dynamic braking has not been attempted before.

In this paper a sincere attempt has been to study and find the finite element solution of the two-dimensional heat conduction in cylindrical polar co-ordinates. The temperature distribution in the $r-\theta$ϴ plane is determined by taking a strip of unit thickness in the stator bound by planes at mid-slot and mid-tooth divided into 30 arch-shaped elements and via explicit nature of matrices limits computer usage and provides a solution to the transient heating problem in stator

This method is directly applicable to the study of temperature rise during DC dynamic braking of induction machines. There are various approaches of finite element a method out of which the method of weighted residuals or Galerkin's Method is used is here because of its relative advantages.

Two Dimensional Transient Heat Conduction Problems and its Formulation by Weighted Residual Approach

For a solid in which heat is being generated internally at rate Q watt/mm^3, consideration of conservation of energy produces the general transient heat conduction equation

$$V\nabla^2 T = P_m C_m \frac{\partial T}{\partial t} - Q \tag{1}$$

In cylindrical polar co-ordinates, equation (1) can be expressed as

$$\frac{1}{r}\frac{\partial}{\partial r}\left(V_r r \frac{\partial T}{\partial r}\right) + \frac{V_\theta}{r^2}\frac{\partial^2 T}{\partial \theta^2} + Q - P_m C_m \frac{\partial T}{\partial t} = 0 \tag{2}$$

Where P_m, C_m are the material density and specific heat, V_r, V_Θ are thermal conductivities in the radial, circumferential directions respectively.

The solution of equation (2) can be obtained by assuming the general functional behavior of the dependent field variable in some way so as to approximately satisfy the given differential equation and boundary conditions. Substitution of this approximation into the original differential equation and boundary conditions then results in some error called a residual. This residual is required to vanish in some average sense over the entire solution domain.

The approximate behavior of the potential function within each element is prescribed in terms of their nodal values and some weighting functions N_1, N_2, **……**Such that

$$T = \sum_{i=1,2,\ldots m} N_i T_i \tag{3}$$

The weighting functions are strictly functions of the geometry and are termed shape functions. These shape functions determine the order of the approximating polynomials for the heat conduction problem.

The required equation governing the behavior of an element is given by the expression.

$$\iint_{D^{(e)}} N_i \left[\frac{\partial}{\partial r}\left(V_r \frac{\partial T^{(e)}}{\partial r}\right) + \frac{\partial}{\partial \theta}\left(\frac{V_\theta}{r^2}\frac{\partial T^{(e)}}{\partial \theta}\right) + Q - P_m C_m \frac{\partial T^{(e)}}{\partial t}\right] r\,d\theta\,dr = 0 \tag{4}$$

Equation (4) can be written as

$$\iint_{D^{(e)}} N_i \left[\frac{\partial}{\partial r}\left(V_r \frac{\partial T^{(e)}}{\partial r}\right) + \frac{\partial}{\partial \theta}\left(\frac{V_\theta}{r^2}\frac{\partial T^{(e)}}{\partial \theta}\right) + Q - P_m C_m \frac{(2T^{(e)} - 2T_0^{(e)})}{2\Delta t}\right] r\,d\theta\,dr = 0 \tag{5}$$

Where T_0 is the temperature at the previous point in time and Δt is the time interval.

Equation (5) expresses the desired averaging to the error or residual within the element boundaries, but it does not admit the influence of the boundary. Since we have made no attempt to choose the N_i so as to satisfy the boundary conditions, we must use integration by parts to introduce the influence of the natural boundary conditions.

Arch Shape Functions

Consider the arch-shaped element of Fig.1 formed by circle arcs radii a, b, radii inclined at an angle 2α.

The shape functions can now be defined in terms of a set of non-dimensional co-ordinates by non-dimensionalising the cylindrical polar co-ordinates r, θ using

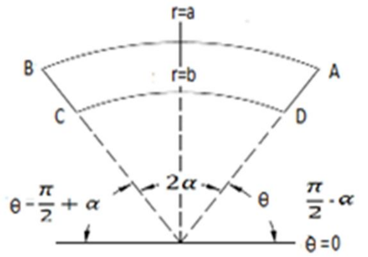

Fig 1. Two-dimensional arch-shaped prism element suitable for discretisation of induction motor stator

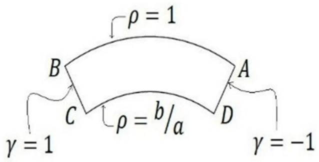

Fig 2. The non-dimensional arch shaped element

$$\rho = \frac{r}{a}; \qquad \gamma = \frac{\theta - \frac{\pi}{2}}{\alpha}$$

The arch element with non-dimensional co-ordinates is shown in Fig. 2.
The temperature at any point within the element be given in terms of its nodal temperatures by

$$T = T_A N_A + T_B N_B + T_C N_C + T_D N_D \qquad (6)$$

Where the N's are shape functions chosen as follows:

$$N_A = \frac{(\rho - \frac{b}{a})(\gamma - 1)}{-2(1-\frac{b}{a})}; \quad N_C = \frac{(\rho - 1)(\gamma + 1)}{-2(1-\frac{b}{a})} \quad N_B = \frac{(\rho - \frac{b}{a})(\gamma + 1)}{2(1-\frac{b}{a})}; \quad N_D = \frac{(\rho - 1)(\gamma - 1)}{2(1-\frac{b}{a})} \qquad (7)$$

Boundary Conditions

The boundary conditions may be written in terms of $\frac{\partial T}{\partial n}$; the temperature gradient normal to the surface. Mid-slot surface $\frac{\partial T}{\partial n_s} = 0$; mid- tooth surface $\frac{\partial T}{\partial n_t} = 0$; Air gap surface $h\,(T-T_{AG}) = -Vr\frac{\partial T}{\partial n_{AG}}$; Where, T= Surface temperature and T_{AG} = Air gap gas temperature.

Back-of-core surface, $h\left(T - T_{BC}\right) = -V_r \frac{\partial T}{\partial n_{BC}}$

Where, T_{BC} = Back of core gas temperature.

Approximate Numeric Form

The heat-flow equation may be formulated in Galerkin's form, the solution being obtained by specializing the general functional form to a particular function, which then becomes the approximate solution sought.

$$\iint_{D^{(e)}} N_i \frac{\partial}{\partial r}\left(V_r \frac{\partial T^{(e)}}{\partial r}\right) r\,dr\,d\theta = \int_{S_2^{(e)}} V_r \frac{\partial T^{(e)}}{\partial r} n_r N_i\, d\Sigma - \iint_{r,\theta} V_r \frac{\partial T^{(e)}}{\partial r} \frac{\partial N_i}{\partial r} r\, d\theta\, dr \qquad (8)$$

Where n_r is the r component of the unit normal to the boundary and $d\Sigma$ is a differential arc length along the boundary. Equation (5) takes the form

$$- \iint_{D^{(e)}} \left(V_r \frac{\partial T^{(e)}}{\partial r}\frac{\partial N_i}{\partial r} + \frac{1}{r^2} V_\theta \frac{\partial T^{(e)}}{\partial \theta}\frac{\partial N_i}{\partial \theta}\right) rd\theta dr$$

$$+ \iint_{D^{(e)}} (N_i Q)\, rdrd\theta$$

$$- \iint_{D^{(e)}} \frac{P_m C_m}{2\Delta t}(2T^{(e)} N_i - 2T_0 N_i)\, rdrd\theta +$$

$$\int_{S_2^{(e)}} \left(V_r \frac{\partial T^{(e)}}{\partial r} n_r + \frac{1}{r^2} V_\theta \frac{\partial T^{(e)}}{\partial \theta} n_\theta) N_i d\sum^{(e)} \right) = 0 \qquad (9)$$

The surface integral (boundary residual) now enables us to introduce the natural boundary conditions. Equation (9) can be written with respect to the nodal temperatures as

$$0 = \iint_{r,\theta} \left[V_r \frac{\partial T^{(e)}}{\partial r}\frac{\partial}{\partial T_i}\left(\frac{\partial T^{(e)}}{\partial r}\right) + \frac{1}{r^2} V_\theta \frac{\partial T^{(e)}}{\partial \theta}\frac{\partial}{\partial T_i}\left(\frac{\partial T^{(e)}}{\partial \theta}\right) - QN_i \right] rdrd\theta$$

$$+ \frac{P_m C_m}{2\Delta t} \iint_{r,\theta} [2\lfloor N \rfloor \{T\}^{(e)} N_i - 2T_0 N_i]\, rdrd\theta +$$

$$\int_{S_2^{(e)}} (h\lfloor N \rfloor \{T\}^{(e)} N_i - hT_\infty N_i) d\Sigma^{(e)} \qquad (10)$$

for i=A, B,C,D

There are four such equations as (10) for the four vertices of the element. These equations when evaluated lead to the matrix equation

$$[[S_R]+[S_\theta]+[S_T]+[S_H]][T] = [S_T][T_0]+[R]+[S_C] \qquad (11)$$

Where, $[S_R]$, $[S_\theta]$ are symmetric co efficient matrices, $[S_T]$ is the heat capacity matrix, $[S_H]$ is the heat convection matrix**,** $[T]$ is the column vector of unknown temperatures**,** $[R]$ is the forcing function (heat source) vector, $[S_C]$ is the column vector of heat convection, $[T_0]$ is the column vector of unknown (previous point in time) temperatures.

Discretized Model for Fem Application

The stator of an induction motor being static is prone to high temperature and the temperature distribution of the stator only is computed here. The hottest spot in the stator is generally in the copper coils. Thermal conductivity of copper and insulation are taken together for calculation. As the temperature is maximum at the central plane, the temperature distribution in the plane can be determined approximately by taking this as a two dimensional r-θ problem with the following assumptions stated here under.

1) The temperature in the strip of unit thickness on the central axis is assumed to be fixed axially i.e no axial flow of heat is assumed in the central plane. This assumption is permissible because in the central plane where the temperature distribution is maximum, the temperature gradient in the axial direction is zero.
2) The convection is taken care of only at the cylindrical surfaces neglecting the convection at the end surfaces. Because of this assumption the temperature calculated in the central plane will be slightly higher than the actual.

In case of transient stator heating during DC Dynamic Braking i.e when DC is injected in the stator of an induction motor, the transient analysis procedure is able to provide an estimate of the temperatures throughout the volume of the stator during the transient period. Assuming that the motor is running initially under steady state condition with small amount of copper loss in the coil slots the equivalent AC current distribution in the stator during DC injection is calculated by equating the resultant amplitude of m.m.f produced by the DC and AC currents respectively. The temperatures within the volume of the stator are calculated at all nodal points for time intervals which ostensibly depend on the magnitude of equivalent AC current, cogently speaking on the amount of DC injected in the stator. In this analysis because of symmetry the two dimensional domain in

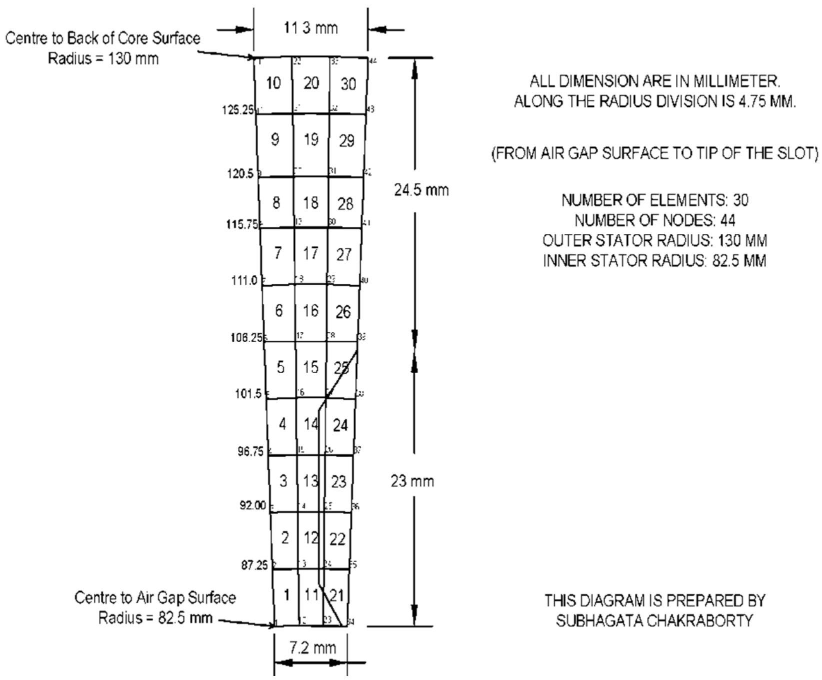

Fig 3. Slice of armature iron and winding bounded by planes at mid-slot and mid-tooth divided into arch-shaped finite elements

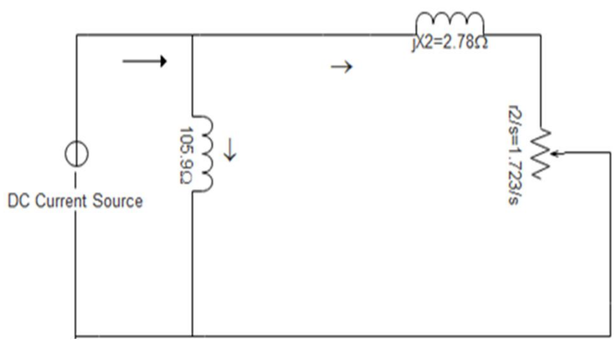

Fig. 4. Equivalent circuit of Induction Motor during DC Dynamic Braking

cylindrical polar co-ordinate of core iron and winding, chosen for modelling of the problem and the geometry is bounded by planes passing through the mid-tooth and mid-slot, which are divided into finite elements. Arch shaped elements are used throughout the solution region. The angular difference between mid-slot and mid-tooth is denoted by 6α which is given as 5° (0.0872665 radian). Hence α turns out to be 0.01454 radian.

Calculation of Heat Losses

Heat losses in the tooth and yoke of the core are best on calculated magnetic flux densities (1.55 Wb/m^2 and 1.14 Wb /m^2 respectively) in these regions, tooth flux lines are mostly radial while the yoke flux lines are mostly circumferential. The grain orientation of the core punching in these two directions influences the heating for a given flux density. Copper losses in the windings are determined on the length as well as area required for the conductors in the slot.

The motor is braked for different values of direct current which is injected in the stator of the motor. Direct current range begins from 50 amps follows a step difference of 5 amps to reach 25 amps finally. The motor is braked from steady state condition after injection of these currents and the temperature rise during this braking phenomena is analyzed.

During DC dynamic braking in a delta connected induction motor following the stator connection illustrated in fig .5, the equivalent AC current to the applied DC follows the formula

$$I_{ac} = \frac{\sqrt{2}}{3} I_{dc} = 0.471 \times I_{dc}$$

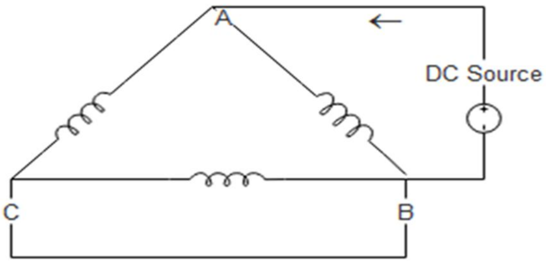

Fig. 5. Stator Connection Used for DC Dynamic Braking (Delta)

The maximum braking torque is given by

$$T_{\max.b} = \frac{3}{\omega_s}\frac{I_{ac}^2 X_M^2}{2(X_M + x_2)} = K \times I_{ac}^2$$

Where K= constant for a machine

$$= \frac{3}{\omega_s}\frac{X_M^2}{2(X_M + x_2)} = \frac{3\times 105.9^2}{2\times 157\times(2.78+105.9)} = 0.985$$

Here for 11 Kw Machine as per design, X_m=105.9 Ω & X_2=2.78 Ω
The retardation time (braking time) is given by

$$t_b = \frac{J\omega_s}{2T_{\max.b}}\left[\frac{1-s^2}{2s_{\max}} + s_{\max}\ln\frac{1}{s}\right]$$

Since the motor is braked from steady state speed every time we inject varying values of DC current into the stator of the AC machine so, slip = s=0.04.
Also, s_{max}=0.308 & J= moment of inertia of load = 10 kg/m^2.

Therefore,

$$t_b = \frac{10\times 157}{2T_{\max.b}}\left[\frac{1-0.04^2}{2\times 0.308} + 0.308\ln\frac{1}{0.04}\right] = \frac{4097.7}{2T_{\max.b}}$$

Copper loss in the stator $= 3\times I_{ac}^2 \times R_1$

Copper loss in the stator per unit volume $= \dfrac{3\times I_{ac}^2 \times R_1}{Volume}$

To calculate stator current at different direct current this is injected in the stator of the motor. Thus the stator copper losses and the time required for dynamic braking calculated and tabulated in Table 1.

Table 1. Different Values of Injected D.C Currents and its Equivalent A.C Counterpart along with its Corresponding Stator Copper Loss/Slot/ Unit Volume and Time Required for D.C Dynamic Braking

Injected DC Current (Amps)I_{DC}	Equivalent AC Current (Amps) I_{ac}	Stator copper loss/slot/ unit volume (Watt/ m^2 °C)	Braking Time (t_b) (seconds)
50	23.57	0.001285	3.73
45	21.21	0.001041	4.62
40	18.84	0.000821	5.86
35	16.48	0.000628	7.66
30	14.13	0.000462	10.418
25	11.77	0.000320	15.02

Convective Heat Transfer Co-Efficient [7,15]

Two separate values of convective heat transfer co-efficient have been taken for the cylindrical curved surface over the stator frame and cylindrical air gap surface. The natural convection heat transfer co-efficient on cylindrical curved surface over the stator frame is taken as h= 5.25 Watt/ m^2 °C.

The heat transfer co-efficient on forced convection for turbulent flow in cylindrical air gap surface is taken as h=60.16 Watt/m^2 °C

Thermal Constants [7,9]

For a transient problem in two dimensions, the following properties are taken for each different element material.

Thermal conductivity, radial direction, V_r Watt/m °C

Thermal conductivity, circumferential direction, V_θ Watt/m °C

Material Density, P_m kg / m^3

Material specific heat, C_m watt sec / kg °C

Table 2. Typical Set of Material Properties for Induction Motor stator

	Magnetic Steel Wedge	Copper & Insulation
V_r	33.070	2.007
V_θ	0.8260	1.062
P_m	7.86120	8.9684
C_m	523.589	385.361

Solutions are done for the two-dimensional stator structure with maximum permissible temperature and then calculating the heat transfer co-efficient at the mean of the temperatures as tabulated below in TABLE 3.

Table 3. Typical Set of Material Properties for Induction Motor stator

Node Number	Temperature For I_{DC}=50 amps	Temperature For I_{DC} =45 amps	Temperature For I_{DC} =40 amps	Temperature For I_{DC} =35 amps	Temperature For I_{DC} =30 amps	Temperature For I_{DC} =25 amps
24	126.636 °C	126.207 °C	126.161 °C	126.089 °C	125.982 °C	125.813 °C
25	127.530 °C	127.493 °C	127.438 °C	127.357 °C	127.239 °C	127.357 °C
26	128.538 °C	128.316 °C	128.256 °C	128.256 °C	128.051 °C	127.874 °C
27	128.763 °C	128.720 °C	128.720 °C	128.587 °C	128.483 °C	128.334 °C
28	128.698 °C	128.682 °C	128.658 °C	128.622 °C	128.570 °C	128.488 °C
35	127.515 °C	127.401 °C	127.257 °C	127.257 °C	126.846 °C	126.536 °C
36	129.020 °C	128.904 °C	128.756 °C	128.565 °C	128.319 °C	127.987 °C
37	129.990 °C	129.878 °C	129.729 °C	129.534 °C	129.280 °C	128.937 °C
38	130.644 °C	130.495 °C	130.310 °C	130.082 °C	129.805 °C	129.456 °C
39	129.273 °C	129.249 °C	129.210 °C	129.169 °C	129.118 °C	129.47 °C

Conclusion

The two-dimensional finite element transient analysis of the stator of an induction motor provides us with an opportunity to study the thermal profile of the stators of various ratings and types of induction motor. Together with the explicitly derived system equations as well as half bandwidth nature of symmetric matrices facilitate the solution of large problems.

A new two dimensional finite element procedure in cylindrical polar co-ordinate using the arch shaped element functions has been used to find the approximate solution of temperature in different parts of a squirrel cage induction motor during braking via Direct Current Injection in the stator of the aforesaid induction motor. Though the above method provides approximated results but the cost effectiveness and quick nature establishes it as an effective method to obtain the visual picture of the thermal profile of the stator of an induction machine. The table which is given here under.

1) Contain new, useable, and fully described information. For example, a specimen's chemical composition need not be reported if the main purpose of a paper is to introduce a new measurement technique. Authors should expect to be challenged by reviewers if the results are not supported by adequate data and critical details.
2) Papers that describe ongoing work or announce the latest technical achievement, which are suitable for presentation at a professional conference, may not be appropriate for publication.

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